Pannellum Equirectangular Projection Reference

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Let a coordinate system be defined with the origin at the focal point, the x-axis parallel to the camera's horizontal axis with positive to the right as viewed from behind the camera, the y-axis parallel to the camera's vertical axis with positive upward as viewed from behind the camera, and the z-axis be perpendicular to these two axes with positive extending away from the camera.[1] Let λ represent the panorama's longitude, ϕ the panorama's latitude, θ the latitudinal offset, ψ the longitudinal offset, and f the focal length. Let $\mathbf{a}$ represent a point's position vector, $\mathrm{R}_x(\theta)$ the latitudinal rotation matrix, $\mathbf{d}$ a point's position vector in the camera's reference frame, x and y a point's position on the image plane, and p_x and p_y a point's position on an equirectangular projection.

$$\mathbf{a} = \begin{bmatrix} \mathbf{a}_x \\ \mathbf{a}_y \\ \mathbf{a}_z \end{bmatrix} = \begin{bmatrix} \sin(\lambda)\cos(\phi) \\ \sin(\phi) \\ \cos(\phi)\cos(\lambda) \end{bmatrix} \tag{1}$$

$$\mathrm{R}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{2}$$

With a point's position vector (1) and the latitudinal rotation matrix (2), one can calculate a point's position in the camera's reference frame.

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}_x \\ \mathbf{d}_y \\ \mathbf{d}_z \end{bmatrix} = \mathrm{R}_x(\theta)\mathbf{a} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \sin(\lambda)\cos(\phi) \\ \sin(\phi) \\ \cos(\phi)\cos(\lambda) \end{bmatrix} = \begin{bmatrix} \sin(\lambda)\cos(\phi) \\ \sin(\phi)\cos(\theta) - \cos(\phi)\cos(\lambda)\sin(\theta) \\ \sin(\phi)\sin(\theta) + \cos(\phi)\cos(\lambda)\cos(\theta) \end{bmatrix} \tag{3}$$

Using the pinhole camera model, one can then calculate positions on the image plane.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{f}{\mathbf{d}_z} \begin{bmatrix} \mathbf{d}_x \\ \mathbf{d}_y \end{bmatrix} = \frac{f}{\sin(\phi)\sin(\theta) + \cos(\phi)\cos(\lambda)\cos(\theta)} \begin{bmatrix} \sin(\lambda)\cos(\phi) \\ \sin(\phi)\cos(\theta) - \cos(\phi)\cos(\lambda)\sin(\theta) \end{bmatrix} \tag{4}$$

$$\frac{x}{f} = \frac{\sin(\lambda)\cos(\phi)}{\sin(\phi)\sin(\theta) + \cos(\phi)\cos(\lambda)\cos(\theta)} \tag{5}$$

$$\frac{y}{f} = \frac{\sin(\phi)\cos(\theta) - \cos(\phi)\cos(\lambda)\sin(\theta)}{\sin(\phi)\sin(\theta) + \cos(\phi)\cos(\lambda)\cos(\theta)} \tag{6}$$

Solving the system of (5) and (6) for λ and ϕ and adding the longitudinal offset, one is able to find a point's position on the unit sphere from its position on the image plane.

$$\lambda = \tan^{-1}\left(\frac{x}{\sqrt{x^2 + (f\cos(\theta) - y\sin(\theta))^2}}, \frac{f\cos(\theta) - y\sin(\theta)}{\sqrt{x^2 + (f\cos(\theta) - y\sin(\theta))^2}} \right) + \psi \tag{7}$$

$$\phi = \tan^{-1}\left(\frac{y\cos(\theta) + f\sin(\theta)}{\sqrt{x^2 + (f\cos(\theta) - y\sin(\theta))^2}} \right) \tag{8}$$

Once a point's position on the unit sphere is known, it is trivial to find its position on an equirectangular projection.

$$p_x = \frac{\lambda}{\pi} \tag{9}$$

$$p_y = \frac{\phi}{\frac{\pi}{2}} \tag{10}$$

[1] Note that this is a left-handed coordinate system.